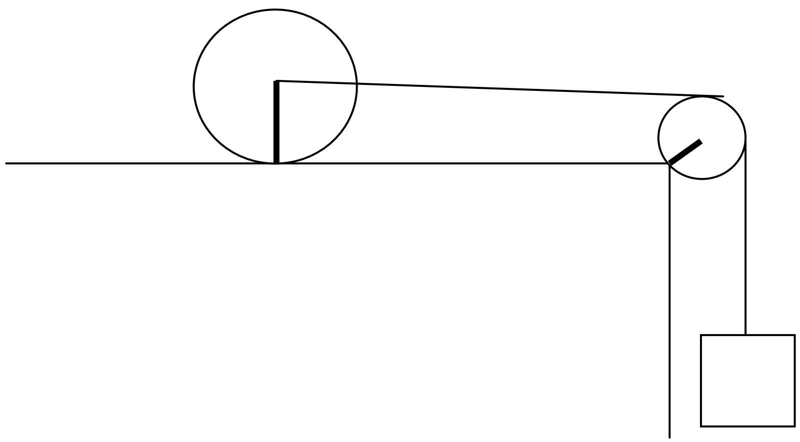
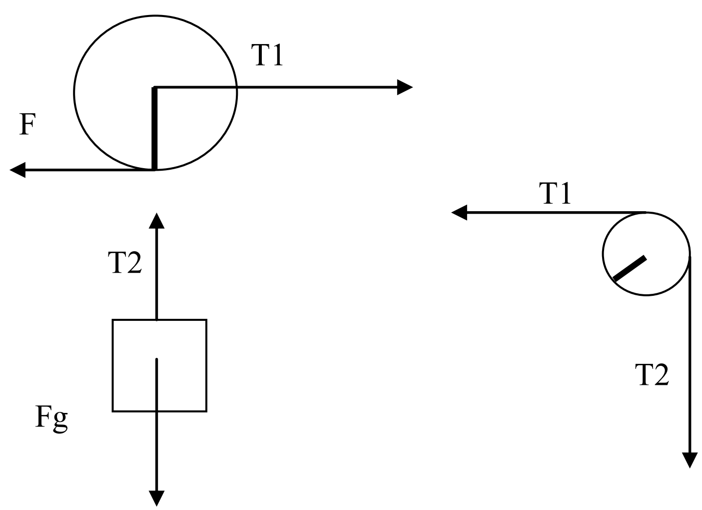
Problem 10.83 Solution by Aaron Epstein & Matt Maloney

The Problem



10.83 A uniform, solid cylinder with mass M and radius 2R rests on a horizontal tabletop. A string is attached by a yoke to a frictionless axle through the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley mass M and radius R that is mounted on a frictionless axle through its center. A block of mass M is suspended from the free end of the string. The string doesn't slip over the pulley surface, and the cylinder rolls without slipping on the table top. Find the magnitude of the acceleration of the block after the system is released from rest.

Step 1: Newton's Law Diagrams



- Solid cylinder of mass M and radius 2R has a positive acceleration in the direction of T1
- The solid cylinder has a torque exerted on it by F: static friction
- The hanging block of mass M has a positive acceleration in the direction of Fg (gravity)
- The pulley has torque exerted on it by T2 and T1, and has a positive angular acceleration in the clockwise direction

Step 2: Setting up equations and math

Block

$\sum F_y = Ma$

$F_g - T_2 = Ma$

$Mg - T_2 = Ma$

$T_2 = M(g - a)$

Pulley

$\sum \mathrm{T} = I\alpha$

$T_2R - T_1R = \frac{1}{2}MR^2 \times \frac{a}{R}$

$T_2R - T_1R = \frac{1}{2}MRa$

$T_2 - T_1 = \frac{1}{2}Ma$

Cylinder

$\sum \mathrm{T} = I\alpha$

$f \times 2R = \frac{1}{2}M(2R)^2 \times \frac{a}{2R}$

$f \times 2R = \frac{1}{2}Ma(2R)$

$f = \frac{1}{2}Ma$

$\sum F_x = Ma$

$T_1 - f = Ma$

$T_1 - \frac{1}{2}Ma = Ma$

$T_1 = \frac{3}{2}Ma$

- Using our diagrams from above, we can set up Newton's Second Law equations for the Block, Pulley and Cylinder
- Block
  - Simple Newton's Second Law equation using the fact that the direction of the acceleration is downwards
- Pulley
  - Use Newton's Second Law for rotation
  - Sum of the torques comes from the two tensions
  - Angular acceleration is clockwise
- Cylinder
  - Need to use both Newton's Second Law and Newton's Second Law for rotation
  - Static friction between the cylinder and the table exerts torque on the cylinder
  - Angular acceleration is clockwise, acceleration is towards the pulley

Step 3: Solve

Equations

$T_2 - T_1 = \frac{1}{2}Ma$

$T_1 = \frac{3}{2}Ma$

$T_2 = M(g - a)$

Finding the Magnitude of the Acceleration

$T_2 - T_1 = \frac{1}{2}Ma$

$M(g - a) - \frac{3}{2}Ma = \frac{1}{2}Ma$

$g - a - \frac{3}{2}a = \frac{1}{2}a$

$a = \frac{g}{3}$

- We will now use the equations we got on the previous page to solve for the magnitude of the acceleration of the suspended block when released from rest
- Plug in our values for T1 and T2 into our equation for the pulley
- Solve to find a
- a = g/3